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Self Shielded Cross-Section Evaluation through Sub Group Method with Improved Resonance Interference Treatment

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1. Introduction

The subgroup method has an advantage of straight-forward determination of self shielded resonance cross-sections without resorting to the intermediate calculation of Dancoff factors [1, 2]. Therefore it has the applicability for the cases of arbitrary geometry or direct whole-core transport calculation. Conventionally in subgroup method the subgroup data is generated without considering resonance interference and is therefore included at the use of subgroup data.

A modification in subgroup method to consider resonance interference explicitly in more consistent way has been proposed in this study. Owing to the fact that these self-shielded cross-sections in interference term is also lethargy dependent, it can be converted to subgroup level dependent self-shielded cross-sections. The proposed method is implemented in 3-D whole core transport lattice code nTRACER .

2. The Subgroup Method

With the intermediate resonance approximation in an infinite homogeneous medium, the slowing down equation is expressed as [3]

$$\hat{\Omega}\nabla\varphi(u)+\sum_i \Sigma_i(u)\varphi(u)=\sum_i \lambda_i\Sigma_{pi}+\sum_i (1-\lambda_i)\Sigma_{si}^{k}(u)\varphi(u) \tag{1}$$

The solution of Eq. (1) with single resonance isotope in terms of the background cross-section, defined as $\sigma_b=\frac{1}{N_R}\sum_i N_i\lambda_i\sigma_{pi}$ is

$$\varphi(u)=\frac{\sigma_b}{\sigma_a(u)+\sigma_b} \tag{2}$$

Where λ is intermediate resonance parameter and σ_p represents macroscopic potential cross-section of the resonance isotope respectively.

The effective cross-sections for the broad group can be determined in terms of subgroup weights (ω_n) and levels (σ_n) using Eq. (3)

$$\sigma_{ai}=\frac{\sum_n \omega_n\sigma_n\frac{\sigma_{bn}}{\sigma_n+\sigma_{bn}}}{1-\sum_n \omega_n\frac{\sigma_{bn}}{\sigma_n+\sigma_{bn}}} \tag{3}$$

2.1 Conventional Resonance Interaction Treatment

Flux of Eq. (2) can be modified in the presence of more than one resonance isotopes in the form given in Eq. (4). The interaction among the resonance isotopes is incorporated through the inclusion of self shielded cross-section term $\sigma_{xi}(u)$ in the non interfered subgroup flux. This is to include the flux dips due to presence of other resonance isotopes [4].

$$\varphi(u)=\frac{\lambda\Sigma_p}{\sum_i N_i\sigma_{ai}(u)+\lambda\Sigma_p}=1-\sum_i\frac{N_i\sigma_{ai}(u)}{\Sigma(u)+\lambda\Sigma_p} \tag{4}$$

The self shielded cross-section can be given by Eq. (5)

$$\sigma_{ai}=\frac{\sum_n \omega_{ni}\sigma_{ni}\frac{\sigma_{bni}}{\sigma_{ni}+\tilde{\sigma}_{xi}+\sigma_{bni}}}{1-\sum_j\sum_n \omega_{nj}\frac{\sigma_{bnj}}{\sigma_{nj}+\tilde{\sigma}_{xj}+\sigma_{bnj}}} \tag{5}$$

Where

$$\sigma_{xi}(u)=\frac{1}{N_i}\sum_{j\neq i} N_j\sigma_{aj} \tag{6}$$

2.2 Modified Resonance Interaction Methodology and Interference Levels

The lethargy dependence of the flux in Eq. (2) and (4) is uniquely through $\sigma_a(u)$. Therefore the lethargy dependence in the integrands of Eq. (7) is also uniquely through $\sigma_a(u)$ and it allows to replace the integration variable u by σ_a and to approximate the integrals by quadratures in σ_a

$$\sigma_x=\frac{\int_{\Delta u}\sigma_x(u)\varphi(u)du}{\int_{\Delta u}\varphi(u)du} \tag{7}$$

In the conventional subgroup interference treatment the self shielded cross-section term by Eq. (6) is used with the approximation that $\sigma_{xi}(u)\rightarrow\sigma_{xi}$. It enforces a constant contribution of interference term from other isotopes in each subgroup. However in the spirit of subgroup formalism, the interfered self shielded scalar flux in Eq. (5) should include subgroup level dependence to be more consistent with the lethargy dependency of Eq. (4). Therefore the contribution from other resonance isotope will be consistent in each subgroup through subgroup weights.

$$\sigma_{ai}=\frac{\sum_n \omega_{ni}\sigma_{ni}\frac{\sigma_{bni}}{\sigma_{ni}+\sigma'_{ni}+\sigma_{bni}}}{1-\sum_j\sum_n \omega_{nj}\frac{\sigma_{bnj}}{\sigma_{nj}+\sigma'_{nj}+\sigma_{bnj}}} \tag{8}$$

where σ'_{ni} can be obtained from σ_{ai} using level dependent correction factors f_{nij} as

$$\sigma'_{ni}=\frac{1}{N_i}\sum_{j\geq i} N_j f_{nij}\sigma_{aj} \tag{9}$$

These correction factors can be obtained for each isotope by comparing numerator A_{ni} and denominators B_{ni} of Eq. (3) with that of Eq. (8) for each subgroup to conserve the self shielded cross-section i.e.

$$\frac{\omega_{ni}\sigma_{ni}\dfrac{\sigma_{bni}n_{\max}j_{\max}}{\sigma_{ni}+\sigma'_{nj}+\sigma_{bni}}}{1-\omega_{nj}\dfrac{\sigma_{ni}n_{\max}j_{\max}}{\sigma_{nj}+\sigma'_{nj}+\sigma_{bni}}}=\frac{A_{ni}}{B_{ni}} \tag{10}$$

Where n_{max} and j_{max} represent the total number of subgroups and resonance isotopes in the mixture. Eq. (10) can be solved for correction factor and the simple algebra leads to the following results.

$$f_{nij}=\frac{(j_{\max}-1)\beta_{ni}}{\dfrac{N_j}{N_i}\sigma_{aj}} \tag{11}$$

and

$$f_{nii}=\frac{\beta_{nj}}{\dfrac{N_i}{N_j}\sigma_{aj}}\left[\frac{\alpha_{nj}\sigma_{bni}}{\beta_{nj}\sigma_{bni}(2-B_{ni})-A_{ni}}-1\right],\ i,j=1,2 \tag{12}$$

where $\alpha_{ni}=\omega_{ni}\sigma_{ni}n_{max}$ & $\beta_{ni}=\sigma_{ni}+\sigma_{bni}$ and $\sigma_{aj} \rightarrow \sigma^{itr}{}_{aj}$ is obtained from Eq. (5) through interference iteration.

4. Calculation and Results

Typical fuel pin cell for the pressurized water reactor is selected as the test case for computation of results.

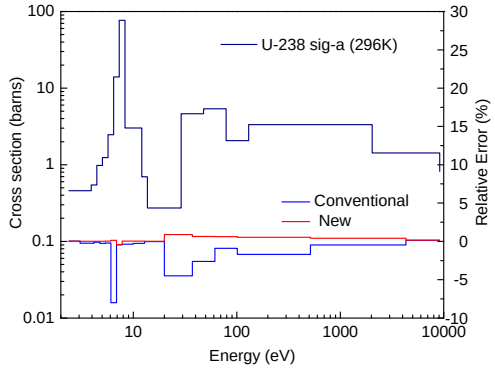

Fig. 1. Comparison of the multi-group absorption cross-sections of ^{235}U at 296K with 4.5 w/o ^{235}U enrichment

The maximum error in absorption cross-sections from the conventional subgroup method for ^{235}U in Fig. 1 is about 25% and for some energy groups with significant differences compared to the reference. However, there are almost no differences in the self-shielded cross-sections between the reference and the new method. The trend is also similar for the ν.fission cross-sections for ^{235}U in Fig. 2 in which the self-shielded cross-sections obtained by the new method are almost identical to the reference ones. The differences obtained between the reference and new resonance interference method for ^{238}U self shielded absorption cross-section estimation is also very small. In order to do performance evaluation of this new method, the final comparison of K_{eff} value computed by nTRACER is made with MCNPX using ENDF/B VII.1. The results are provided in Fig. 3 which indicates improved K_{eff} by the new method as compared to the conventional method of resonance interference interaction at different fuel temperatures.

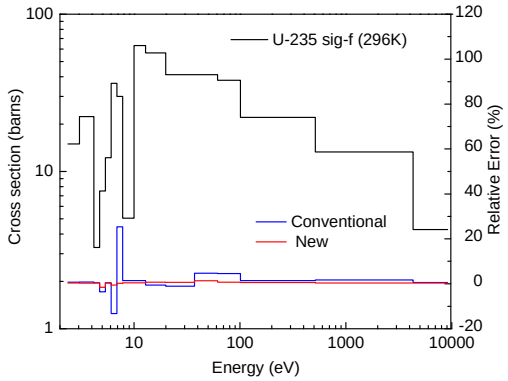

Fig. 2. Comparison of the ν.fission cross-sections of ^{235}U at 296K with 4.5 w/o ^{235}U enrichment.

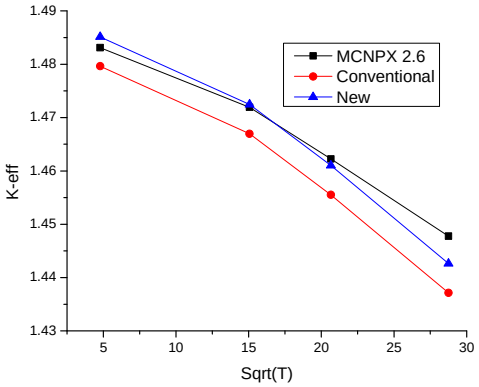

Fig. 3. K_{eff} vs. square root of fuel temperature for typical PWR fuel pin.

5. Conclusions

More consistent method of resonance interference interaction has shown relatively negligible error in self shielded cross-section. This new interference treatment method is investigated at various temperatures and has shown better results regardless of temperature changes of mixture of resonance isotopes mixture.

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